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Quantum light in the turbulent atmosphere

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Nonclassical properties of light propagating through the turbulent atmosphere are studied. We demonstrate by numerical simulation that the probability distribution of the transmission coefficient, which characterizes the effects of the atmosphere on the quantum state of light, can be reconstructed by homodyne detection. Nonclassical photon-statistics and, more generally, nonclassical Glauber-Sudarshan functions appear to be more robust against turbulence for weak light fields rather than for bright ones.

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Nonclassical properties of quantum light have been of great interest from the viewpoint of fundamentals of quantum physics and for a variety of applications, such as quantum information processing and quantum metrology. Special knowledge on the propagation of quantum light through the turbulent atmosphere is required in the context of implementations of quantum cryptography for communication channels between earth-based stations [1] and between satellites and Earth-based stations [2]. The theory is well established for the propagation of classical light through the atmosphere, see, e.g., [3], including phenomena such as beam wander, beam spreading, scintillations, degradation of spatial coherence, and others. However, nonclassical properties, such as sub-Poissonian statistics of photocounts [4], quadrature squeezing [5], non-positivity of the Glauber-Sudarshan P function [6, 7], and entanglement [8] have been little studied in the context of the propagation of light through the turbulent atmosphere.

Due to the occurrence of random fluctuations of the refractive index, the quantum state of light after transmission through the atmosphere cannot be presented by a single-mode density operator, in terms of neither monochromatic nor nonmonochromatic modes. The photocounting statistics of a combination of scattered modes has been studied by a random modulation of the intensity [9, 10]. This model has been further improved [11]. The technique of the photon wave-function allows one to consider special cases of single-photon [12] and two-photon [13] states. Another approach, which describes single-photon states, is presented in Ref. [14].

In the present contribution we deal with the effects of an atmospheric transmission channel on any quantum state of light. General expressions for the quantum state after transmission are derived. Based on balanced homodyne detection, one may reconstruct the statistical distribution of the transmission coefficient through the atmosphere. By repeated reconstruction of the statistical distribution, one can significantly reduce the atmospheric noise effects on the quantum state of light.

For dealing with continuous-variable quantum states, we are considering the experimental setup in Fig. 1. The light from a source is transmitted through the atmosphere and collected by a telescope (or some other device). Subsequently, a balanced homodyne detection setup is used to filter out the desired nonmonochromatic mode from other modes and background radiation by an appropriate local oscillator, for details see [15, 16, 17]. The remote local oscillator can be synchronized with the source field by, e.g., the technique of the optical frequency comb [18]. In the limit of a strong local oscillator, the difference of photocurrents in the detectors is proportional to the field quadrature of the nonmonochromatic output mode defined by the local-oscillator pulse. Knowledge of the quadrature distributions enables one to get the complete information about the quantum state of the considered output mode. For example, the method of optical homodyne tomography enables one to reconstruct the Wigner function, the photon-number distribution, moments of the radiation field, and the density operator in an arbitrary representation, for a review see [16].

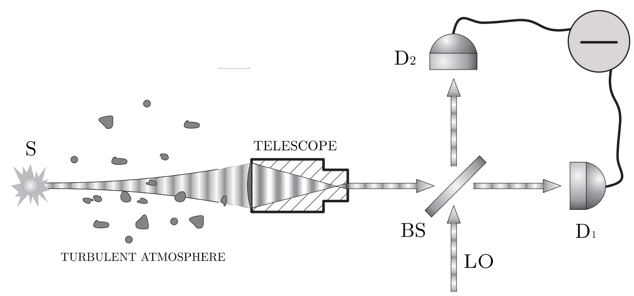

FIG. 1: Homodyne detection of quantum light generated by the source, S, and propagated through the turbulent atmosphere. BS is a 50:50 beam-splitter, D_1 and D_2 are detectors, LO is the local oscillator.

We assume that the propagation of quantum light through the turbulent atmosphere is a linear attenuating process. The corresponding losses are caused by light absorption and scattering as well as by the mode-mismatch

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due to different shapes of the output and local-oscillator pulses. The Glauber-Sudarshan P function [19] of an attenuated nonmonochromatic output mode, $P_T(\alpha)$, is related to the P function of the input mode, $P_{\rm in}(\alpha)$, as (cf., e.g., [20])

$$P_T(\alpha) = \frac{1}{|T|^2} P_{\rm in}\left(\frac{\alpha}{T}\right), \tag{1}$$

where T is the complex transmission coefficient, with $|T|^2 \leq 1$. An important difference between turbulent media and other lossy systems is that the transmission coefficient T is a random variable with fluctuating phase and magnitude. This means that the P function of the output mode, $P_{\rm out}(\alpha)$, is obtained through averaging $P_T(\alpha)$ with the probability distribution of the transmission coefficient (PDTC), $\mathcal{P}(T)$, as

$$P_{\rm out}(\alpha) = \int\limits_{|T|^2\leq 1} \mathrm{d}^2 T\, \mathcal{P}(T) \frac{1}{|T|^2} P_{\rm in}\left(\frac{\alpha}{T}\right). \tag{2}$$

The integration is performed over the circular area, $|T|^2 \leq 1$. This represents the quantum-state input-output relation for light propagated through the turbulent atmosphere.

The explicit form of the PDTC should be obtained from a theory, which considers turbulence properties of the atmosphere as well as specific conditions of the experiment. Since this is a complex problem, we may only consider a simple model for the case of small fluctuations. The corresponding PDTC can be obtained similar to the probability distribution of the intensity modulation in Ref. [9]. For this purpose, we consider a discrete set of turbulent eddies, each of them is characterized by a random transmission coefficient T_k. The total transmission coefficient is $T = \prod_k T_k$. The central limit theorem implies that the PDTC is a two-dimensional distribution, which is log-normal with respect to the magnitude $t = |T|$ and normal with respect to the phase $\varphi = \arg T$,

$$\mathcal{P}(t,\varphi) \approx \frac{1}{2\pi t \sigma_\theta \sigma_\varphi \sqrt{1-s^2}} \times e^{-\frac{1}{2(1-s^2)}\left[\left(\frac{\ln t + \bar{\theta}}{\sigma_\theta}\right)^2 + \left(\frac{\varphi}{\sigma_\varphi}\right)^2 + 2s \frac{\ln t + \bar{\theta}}{\sigma_\theta}\frac{\varphi}{\sigma_\varphi}\right]}. \tag{3}$$

Here, $\bar{\theta}$ and σ_θ are the mean value and the variance, respectively, of $\theta = -\ln t$; σ_φ is the variance of φ; s is the correlation coefficient between θ and φ. Without loss of generality we suppose that the mean value of φ is zero. This form of the PDTC can be used only for $\sigma_\theta \ll \bar{\theta}$ and $\sigma_\varphi \ll 2\pi$. Contrary to the approach based on the random modulation of the intensity [9, 10, 11], we restrict the t-integration to the range $0 \leq t \leq 1$. Accordingly, the φ-integration is restricted to $-\pi \leq \varphi \leq \pi$.

Of course, the given model [Eq. (3)] will not properly describe the turbulence properties of the atmosphere under general conditions. Due to the lack of a general model, it is important to develop a method for the experimental determination of the PDTC. Let the input field be prepared in a coherent state $|\gamma\rangle$. In this case the PDTC can be expressed in terms of the characteristic function $\Phi_{\rm out}(\beta)$, of the P function of the output state as

$$\mathcal{P}(T_r, T_i) = \frac{1}{4} \sum_{n,m=-\infty}^{+\infty} \Phi_{\rm out}\left(\frac{\pi}{2\gamma^*}[m+in]\right) e^{i\pi(mT_i - nT_r)}, \tag{4}$$

where T_r and T_i are the real and imaginary parts of the transmission coefficient, respectively. In an optical homodyning experiment $\Phi_{\rm out}(\beta)$ can be estimated from a sample of N photocounting difference events Δn_j, cf., [7, 16],

$$\Phi_{\rm out}(\beta) = e^{\frac{|\beta|^2}{2}} \frac{1}{N} \sum_{j=1}^{N} \exp\left[i \frac{|\beta|\,\Delta n_j}{r}\right]. \tag{5}$$

The amplitude and the phase of the local oscillator are fixed to be r and $\left(\frac{\pi}{2} - \arg\beta\right)$, respectively.

To demonstrate the practical usefulness of the reconstruction method of the PDTC, we have performed the following simulation. We start with model distribution (3). For simplicity, it is approximated here by a normal distribution in the variables T_r and T_i. In practice, observed data shall be used, so that this assumption does not restrict the applicability of our method. We derive the P function of the output field from Eq. (2) and calculate the photocount-difference distribution by using the corresponding integral transformation, see Ref. [21]. Now we can simulate the measured data and reconstruct by Eqs. (4) and (5) the PDTC. The result of this procedure is shown in Fig. 2, which is in reasonable agreement with the initially chosen PDTC even for a small sample of data. In real experiments, this method yields insight into the true statistics of the turbulent atmosphere.

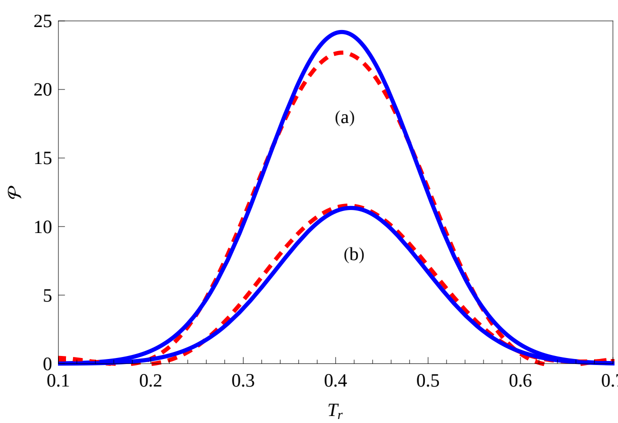

FIG. 2: (Color online) The PDTC is shown for a normal-distribution approximation of Eq. (3) with $\bar{\theta} = 0.9, \sigma_\theta = 0.2, \sigma_\varphi = 0.2, s = 0.01$, $T_i = 0$ (a), 0.1 (b). The solid and the dashed lines are the initially chosen and the reconstructed (from only 5×10^3 sampling events) PDTC, respectively.

Alternatively, the PDTC can also be characterized by its statistical moments. Consider the matrix of normal-

ordered moments of the photon annihilation (creation) operator $\hat{a}$ ($\hat{a}^\dagger$) for the source field,

$$M_{nm} = \mathrm{Tr}\left[\hat{\rho}\,\hat{a}^{\dagger\,n}\hat{a}^m\right] \equiv \int_{-\infty}^{+\infty} \mathrm{d}^2\alpha\, P\left(\alpha\right)\alpha^{*n}\alpha^m, \qquad (6)$$

which completely characterizes the quantum state of a light mode, where $\hat{\rho}$ is the density operator. The moments can be measured by balanced homodyne detection [16]. Utilizing the input-output relation [Eq. (2)], one gets the corresponding relation between the normal-ordered moments of the input and output modes,

$$M_{nm}^{\mathrm{out}} = \left\langle T^{*n}T^m\right\rangle M_{nm}^{\mathrm{in}}, \qquad (7)$$

where

$$\left\langle T^{*n}T^m\right\rangle = \int_{|T|^2\leq 1} \mathrm{d}^2T\,\mathcal{P}\left(T\right)T^{*n}T^m \qquad (8)$$

are the moments of the PDTC. From Eq. (7), the moments of the PDTC,

$$\left\langle T^{*n}T^m\right\rangle = \frac{M_{nm}^{\mathrm{out}}}{M_{nm}^{\mathrm{in}}}, \qquad (9)$$

are obtained by measuring the moments of the input and output fields. For example, one can use the input mode in a coherent state $|\gamma\rangle$ such that $M_{nm}^{\mathrm{in}} = \gamma^{*n}\gamma^m$. The obtained moments of the PDTC allow one to determine the nonclassical properties of the output mode, once the corresponding properties of the input mode are known.

It is known [3] that due to the atmospheric winds narrow light beams are randomly deflected and wide beams scintillate within a certain time τ_{atm}. For data accumulation times $\tau_{\mathrm{data}} \gg \tau_{\mathrm{atm}}$ we expect large fluctuations of the transmission coefficient T. Otherwise, for $\tau_{\mathrm{data}} \ll \tau_{\mathrm{atm}}$ the transmission coefficient is not significantly fluctuating during the time τ_{data}. However, for this scenario, $\mathcal{P}\left(T\right)$ is randomly changed between different series of measurements, separated by time intervals $\Delta\tau \gtrsim \tau_{\mathrm{atm}}$. The PDTC and its moments can thus be permanently monitored by using the method proposed above, which works with a small sample of data that can be recorded within short τ_{data} intervals. In this way one can suppress the influence of long-term atmospheric fluctuations on the quantum state of the transmitted light.

Let us consider the transmission of sub-Poissonian light [4] through the turbulent atmosphere. Using Eq. (7), the Mandel parameter (cf., e.g., [20]) of the output field, Q_{out}, can be related to the Mandel parameter of the input field, Q_{in}, as

$$Q_{\mathrm{out}} = \frac{\left\langle\eta^2\right\rangle}{\left\langle\eta\right\rangle}Q_{\mathrm{in}} + \frac{\left\langle\Delta\eta^2\right\rangle}{\left\langle\eta\right\rangle}M_{11}^{\mathrm{in}}, \qquad (10)$$

where $\eta = T^*T$ is the efficiency. The first term resembles the behavior for standard attenuation. The second term is caused by fluctuations of the efficiency η due to the atmospheric turbulence. It is proportional to the mean photon-number of the input field, $M_{11}^{\mathrm{in}} = \left\langle\hat{n}\right\rangle_{\mathrm{in}}$. For states of the input field whose mean photon-number fulfills

$$\left\langle\hat{n}\right\rangle_{\mathrm{in}} > -\frac{\left\langle\eta^2\right\rangle}{\left\langle\Delta\eta^2\right\rangle}Q_{\mathrm{in}}, \qquad (11)$$

the photocounts of the output mode are always super-Poissonian. Hence the nonclassical photon statistics of bright quantum light is destroyed by fluctuations of the magnitude t of the transmission coefficient, but it is not affected by phase noise.

In other cases, the nonclassical properties of bright light can also become sensitive to both phase and magnitude noise. In the most general case a given quantum state is nonclassical if its P function is not positive definite [6]. For weak turbulence, the PDTC has a strong maximum at $T = T_{\mathrm{o}}$. Hence the input-output relation [Eq. (2)] reads in the first-order Laplace approximation as

$$P_{\mathrm{out}}\left(\alpha\right) \approx \int_{-\infty}^{+\infty} \mathrm{d}^2\beta\, \frac{1}{|T_{\mathrm{o}}|^2}P_{\mathrm{in}}\left(\frac{\beta-\gamma}{T_{\mathrm{o}}}\right)\frac{1}{|\gamma|^2}\mathcal{P}\left(\frac{\alpha-\beta}{\gamma}\right), \qquad (12)$$

where $\gamma = \left\langle\hat{a}\right\rangle_{\mathrm{in}}$ is the displacement parameter of the input field and the PDTC is taken in the Gaussian approximation [22]. If the minimum eigenvalue of the covariance matrix of the scaled PDTC in Eq. (12) obeys $\lambda_{\mathrm{min}} \geq 2$, $P_{\mathrm{out}}\left(\alpha\right)$ represents the Husimi-Kano Q function [20] of the displaced input field combined with a Gaussian noise, which is always non-negative. Based on the above approximations, we derive that for any input state with

$$|\gamma| \geq 2e^{\bar{\theta}}\sqrt{\left(\sigma_\theta^2+\sigma_\varphi^2-\sqrt{\left(\sigma_\theta^2-\sigma_\varphi^2\right)^2+4s^2\sigma_\theta^2\sigma_\varphi^2}\right)^{-1}} \qquad (13)$$

the corresponding output state is classical. For simplicity we have considered only real γ and $\bar{\varphi} = 0$, the generalization to complex γ and arbitrary $\bar{\varphi}$ is straightforward.

As an example we consider the P function of displaced single-photon-added thermal states (SPATSs) [7],

$$P_{\mathrm{in}}\left(\alpha\right) = \frac{1}{\pi\bar{n}_{\mathrm{th}}^3}\left(\left(1+\bar{n}_{\mathrm{th}}\right)|\alpha-\gamma|^2-\bar{n}_{\mathrm{th}}\right)e^{-\frac{|\alpha-\gamma|^2}{\bar{n}_{\mathrm{th}}}}, \qquad (14)$$

where $\bar{n}_{\mathrm{th}}$ and γ are the mean number of thermal photons and the coherent displacement amplitude, respectively. We compare the P functions of the displaced SPATS for two cases: (i) the standard attenuation with a fixed transmission coefficient T in Eq. (1), and (ii) transmission through the turbulent atmosphere as described by model (3). The mean values of the transmission coefficient are chosen to be equal in both cases. The standard attenuation does not destroy negativities of the P function. For small displacements, the situations in the cases (i) and (ii) remain similar to each other. However, with

increasing displacements γ the atmospheric turbulence destroys the nonclassical effects, even when they survive for standard attenuation, see Fig. 3.

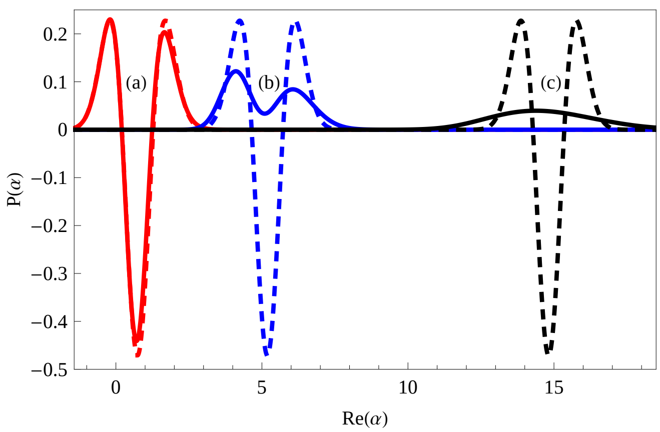

FIG. 3: (Color online) P function of the displaced SPATS, for $\mathrm{Im}\,(\alpha) = 0$, $\bar{n}_{\mathrm{th}} = 1.11$, $\gamma = 1$ (a), 7 (b), 20 (c). The dashed lines show the standard attenuation for $T = e^{-0.3} \approx 0.7408$. The solid lines represent the model (3) for $\bar{\theta} = 0.3$, $\sigma_\theta = 0.1$, $\sigma_\varphi = 0.14$, and $s = 0.01$.

In conclusion, we have studied the effects of atmospheric turbulence on the quantum properties of light. It has been shown that the probability distribution of the transmission coefficient can be experimentally reconstructed by homodyne measurements. Based on such a method, one may predict the general turbulence effects on any quantum state of light. By repeated short-time monitoring of the PDTC, one can suppress the disturbing effects of long-term fluctuations on the quantum state of light. Balanced homodyne detection also allows one to reduce the effects of background radiation, which is useful for quantum communications under day-light conditions. We have shown that the nonclassical effects of bright light fields can be more fragile against turbulence than for the case of weak fields. A nonclassical photon-statistics is destroyed by turbulence if the mean photon number exceeds a critical value. General nonclassicality of the P function in the output channel is sensitive to the mean coherent amplitude of the input radiation.

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